Variables Separable Differential Equations

A differential equation with separable variables can be written in the form

$$\frac{dy}{dx} = f(x)g(y)$$

We solve it by separating the variables. We integrate the left-hand side with respect to y and we integrate the right-hand side with respect to x

$$\int \frac{1}{g(y)}\,dy = \int f(x)\,dx$$

We can find the general solution to a differential equation and we are often asked to write the solution for y explicitly, i.e. $y = f(x)$

e.g. Solve $\frac{dy}{dx} = \frac{sinx}{y^2}$

We separate the variables. We integrate the left-hand side with respect to y and we integrate the right-hand side with respect to x

$$\int y^2\,dy = \int sinx\,dx$$

We only need one arbitrary constant

$$\frac{y^3}{3} = -cosx + c$$

Multiplying an arbitrary constant by 3 still gives an arbitrary constant

$$y^3 = -3cosx + c_1$$

$$y = \sqrt[3]{c_1 - 3cosx}$$

We can also be asked to find a particular solution to a differential equation if we are given some initial conditions

e.g. Solve $\frac{dy}{dx} = \frac{y}{x}$, given that $y(1) = 2$

$$\int \frac{1}{y}\,dy = \int \frac{1}{x}\,dx$$

We can simplify more easily if we think of c as lnk

$$ln|y| = ln|x| + c$$

$$ln|y| = ln|x| + lnk$$

$$ln|y| = ln|kx|$$

$$y = kx$$

When x=1, y=2 , so $2 = k$

$$y = 2x$$