Variables Separable Differential Equations

A differential equation with separable variables can be written in the form

$$\frac{dy}{dx} = f(x)g(y)$$

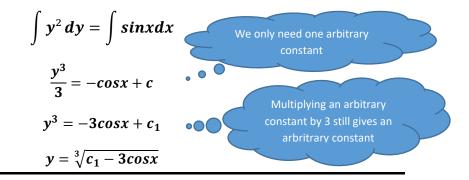
We solve it by separating the variables. We integrate the left-hand side with respect to y and we integrate the right-hand side with respect to x

$$\int \frac{1}{g(y)} dy = \int f(x) dx$$

We can find the general solution to a differential equation and we are often asked to write the solution for y explicitly, i.e. y = f(x)

e.g. Solve
$$\frac{dy}{dx} = \frac{sinx}{y^2}$$

We separate the variables. We integrate the left-hand side with respect to y and we integrate the right-hand side with respect to x



We can also be asked to find a particular solution to a differential equation if we are given some initial conditions

e.g. Solve
$$\frac{dy}{dx} = \frac{y}{x}$$
, given that $y(1) = 2$

$$\int \frac{1}{y} dy = \int \frac{1}{x} dx$$

$$ln|y| = ln|x| + c$$

$$ln|y| = ln|x| + lnk$$

$$ln|y| = ln|kx|$$

$$y = kx$$

When x=1, y=2, so $\mathbf{2} = \mathbf{k}$

$$y = 2x$$



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