## Volume of Revolution

Rotating $y=f(x)$ about the $\mathbf{x}$ axis


The volume generated when the area between the curve $y=f(x)$ and the x axis from $x=a$ and $x=b$ is rotated $2 \pi$ radians about the $\mathbf{x}$ axis

$$
V=\pi \int_{a}^{b} y^{2} d x \text { or } V=\pi \int_{a}^{b}[f(x)]^{2} d x
$$

Rotating $x=f(y)$ about the $\mathbf{y}$ axis


The volume generated when the area between the curve $x=f(y)$ and the y axis from $\mathrm{y}=a$ and $y=b$ is rotated $2 \pi$ radians about the $\mathbf{y}$ axis

$$
V=\pi \int_{a}^{b} x^{2} d y \text { or } V=\pi \int_{a}^{b}[f(y)]^{2} d y
$$

## Rotating region bounded by two graphs

When a region like this is rotated about the $x$ axis, a hollowed out solid is produced



You can find the volume generated under the red line and subtract the volume generated under the green curve


The volume generated when the area between the curve $y=g(x)$ and $y=f(x)$ from $x=a$ and $x=b$ is rotated $2 \pi$ radians about the $\mathbf{x}$ axis

$$
V=\pi \int_{a}^{b}[g(x)]^{2} d x-\pi \int_{a}^{b}[f(x)]^{2} d x
$$

This can be simplified to

$$
V=\pi \int_{a}^{b}\left([g(x)]^{2}-[f(x)]^{2}\right) d x
$$

...and a similar method can be used if the region is rotated about the $y$ axis

