The Binomial Distribution

The binomial distribution is an example of a discrete random variable. It has two parameters **n** (number of trials) and **p** (probability of success of one trial):

$$\boldsymbol{X \sim B(n, p)}.$$

For a situation to be described using a binomial model, the following must be true

- A finite number of trials, n are carried out
- The trials are independent
- The outcome of each trial is deemed either a success or a failure
- The probability, p, of a successful outcome is the same for each trial

We calculate probabilities using the following formula

$$\boldsymbol{P(X = r) = {}^{n}C_{r} \times p^{r} \times (1 - p)^{n-r}}$$

$\boldsymbol{{}^{n}C_{r} = \binom{n}{r}}$ is the combination of selecting ***r*** items from a total of ***n*** items

Often we can use the formula in our graphical calculators:

Binompdf is for finding individual values e.g. $P(X = 3)$

Binomcdf is for finding the probability of an interval e.g. $P(X \geq 3)$

Care needs to be taken to interpret what the question is asking. Writing out the possible outcomes helps to avoid making mistakes:

e.g. $X \sim B(8, 0.3)$

P(exactly 2)	0 1 2 3 4 5 6 7 8	binompdf, x = 2
P(at least 2)	0 1 2 3 4 5 6 7 8	binomcdf, lower = 2, upper = 8
P(less than 3)	0 1 2 3 4 5 6 7 8	binomcdf, lower = 0, upper = 2
$P(2 < X \leq 6)$	0 1 2 3 4 5 6 7 8	binomcdf, lower = 3, upper = 6

The mean or expected value, $E(X) = np$

The variance, $Var(X) = npq$ $q = 1 - p$