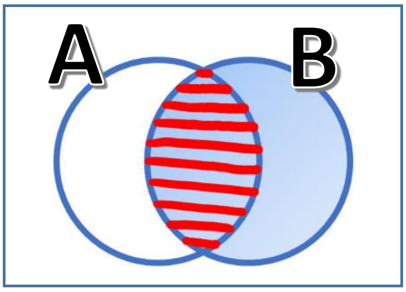
Conditional Probability

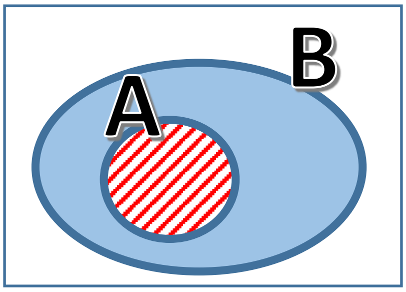
Conditional probability is the probability of an event happening **given that** another event has already happened. The probability of A happening **given that** B has happened is written $P(A/B)$



$$P(A/B) = \frac{P(A \cap B)}{P(B)}$$

Remember that $P(A \cap B)$=P(A **AND** B)

Specific Case - A is a subset of B



$$P(A/B) = \frac{P(A)}{P(B)}$$

An example of this is rolling a die:

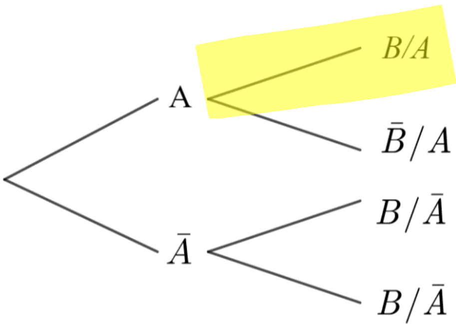
$$P(X \leq 2/X \leq 3) = \frac{2}{3}$$

1 2 3 4 5 6

This situation happens often with questions about probability distributions like the Binomial Distribution and the Normal Distribution.

We can solve questions about conditional probability using **probability trees**

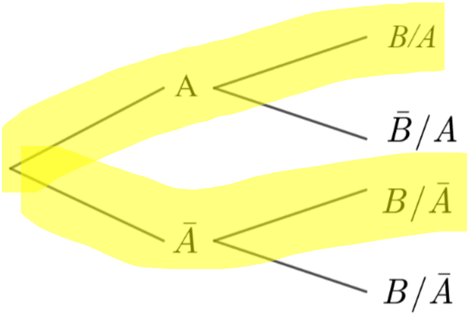
$P(B/A)$ is obvious from the tree diagram



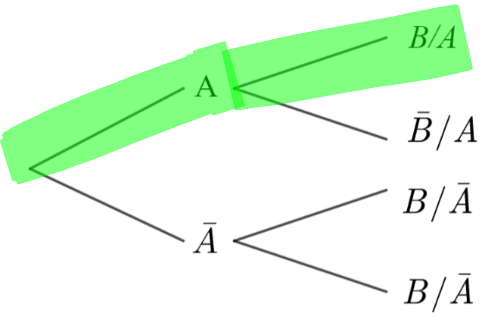
The **reversed case** of $P(A/B)$ is more challenging. There is a formula given below which is called **Bayes' Theorem**, but it looks quite complicated and it is better to split the probability into two parts like this:

$$P(A/B) = \frac{P(A \cap B)}{P(B)}$$

We can find $P(B) = P(A) \times P(B/A) + P(\bar{A}) \times P(B/\bar{A})$



We can find $P(A \cap B) = P(A) \times P(B/A)$



So,

$$P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{P(A) \times P(B/A)}{P(A) \times P(B/A) + P(\bar{A}) \times P(B/\bar{A})}$$

$$P(A/B) = \frac{P(A) \times P(B/A)}{P(A) \times P(B/A) + P(\bar{A}) \times P(B/\bar{A})}$$

This formula is sometimes called **Bayes' Theorem** and is probably the most important theorem in probability. It has many applications, including medicine, where it is used to determine the accuracy of medical tests.