The continuous random variable X has a probability density function given by

$$f(x) = \begin{cases} \text{k} \cdot arcos(x) & \text{-}1 \leq x < 1 \\ 0 & otherwise \end{cases}$$

a) Draw a sketch of ***f***
b) State the mode of X
c) Find $\int arcos(x)dx$
d) Hence find k
e) Find E(x)
f) Find Var(X)

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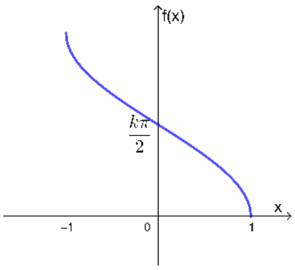
a) $$k \cdot arcos(0) = \frac{\text{k}\pi}{2}$$



b) Mode = -1

c) We can use our graphical calculators for parts d) and e) , but we must work out this integral by hand.

We need to use integration by parts $\int u \cdot \frac{dv}{dx}\, dx \;=\; uv \;-\; \int v \cdot \frac{du}{dx}\, dx$

$$\int 1 \cdot arcos(x)dx$$

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$u = arccosx$ $\qquad \frac{dv}{dx} = 1$

$\frac{du}{dx} = -\frac{1}{\sqrt{1-x^2}}$ $\qquad v = x$

$$= arccosx + \int \frac{x}{\sqrt{1-x^2}}dx$$

$$= arccosx - \frac{1}{2}\int \frac{\text{-}2x}{\sqrt{1-x^2}}dx$$

$$= arccosx - \frac{1}{2}\ln\sqrt{1-x^2} + \text{C}$$

d) For continuous random variables, the total area under the function = 1

$$\int_{-\infty}^{\infty} f(x)dx = 1$$

$$\int_{-1}^{1} k \cdot arcos(x)dx = 1$$

$$\text{k}\left[arccosx - \frac{1}{2}\ln\sqrt{1-x^2}\right]_{-1}^{1} = 1$$

$$k\left(\left[arccos(1) - \frac{1}{2}ln\sqrt{1-1^2}\right] - \left[arccos(-1) - \frac{1}{2}ln\sqrt{1-(-1)^2}\right]\right) = 1$$

$$k\left(\left[0 - \frac{1}{2}ln0\right] - \left[\pi - \frac{1}{2}ln0\right]\right) = 1$$

$$k\pi = 1$$

$$k = \frac{1}{\pi}$$

e)

$$f(x) = \begin{cases} \frac{1}{\pi} \cdot arcos(x) & -1 \le x < 1 \\ 0 & otherwise \end{cases}$$

For continuous random variables,

$$\text{E}(X) = \int_{-\infty}^{\infty} xf(x)dx$$

$$\text{E}(X) = \int_{-1}^{1} \frac{1}{\pi} x \cdot arcos(x)dx$$

Use GDC

$$\text{E}(X) = -0.25$$

g)

$$Var(X) = E(X^2) - [E(X)]^2$$

$$E(X^2) = \int_{-1}^{1} \frac{1}{\pi} x^2 \cdot arcos(x)dx = \frac{1}{3}$$

$$Var(X) = \frac{1}{3} - (-0.25)^2$$

Either of the following answers is acceptable:

$$Var(X) = \frac{13}{48}$$

$$Var(X) \approx 0.271$$