Discrete Random Variables

These are just a way of using more formal language to describe probability events. In the examination, they can be given in 2 different ways: in table form or as a function (it is also useful to recognize graphs).

Let's consider a simple case of flipping 3 coins. In this case, there are 8 equally likely outcomes

HHH , HHT , HTH , HTT , THH , THT , TTH , TTT

Imagine we are interested in the number of heads that we flip. We can either flip 0 or 1 or 2 or 3 heads:

Number of heads	Probability
0	$\frac{1}{8}$
1	$\frac{3}{8}$
2	$\frac{3}{8}$
3	$\frac{1}{8}$

Let ***X*** be the random variable: the number of heads flipped when we flip 3 coins. We can represent this random variable

1) in table form

X	0	1	2	3
P(X = x)	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

2) as a function

$$\mathrm{P}(\mathrm{X} = \mathrm{x}) = \binom{3}{x} \cdot \frac{1}{8} \quad \text{for } x \in \{0, 1, 2, 3\}$$

We can easily find the expected value using the formula

$$E(X) = \sum xP(X = x)$$

In the case above, E(X) = $0 \cdot \frac{1}{8} + 1 \cdot \frac{3}{8} + 2 \cdot \frac{3}{8} + 3 \cdot \frac{1}{8} = 1.5$

We could have used the fact that the function is symmetrical to work this out!

Note, **Fair games** mean that $E(X) = 0$

Exam questions often give the random variable as a function in which you use the fact that probabilities of all possible outcomes add up to 1 to work out an unknown:

$\mathrm{P(X\ =\ x)} = kx$ for $x \in \{1, 2, 3, 4\}$

Find ***k***

$\mathrm{P(X\ =\ 1)} = k \cdot 1$

$\mathrm{P(X\ =\ 2)} = k \cdot 2$

$\mathrm{P(X\ =\ 3)} = k \cdot 3$

$\mathrm{P(X\ =\ 4)} = k \cdot 4$

$$\sum P(X = x) = k + 2k + 3k + 4k = 1$$

$$10k = 1$$

$$k = 0.1$$