A box contains 25 tickets. $\boldsymbol{x}$ tickets are gold, the rest are silver. **Two** tickets are selected at random.

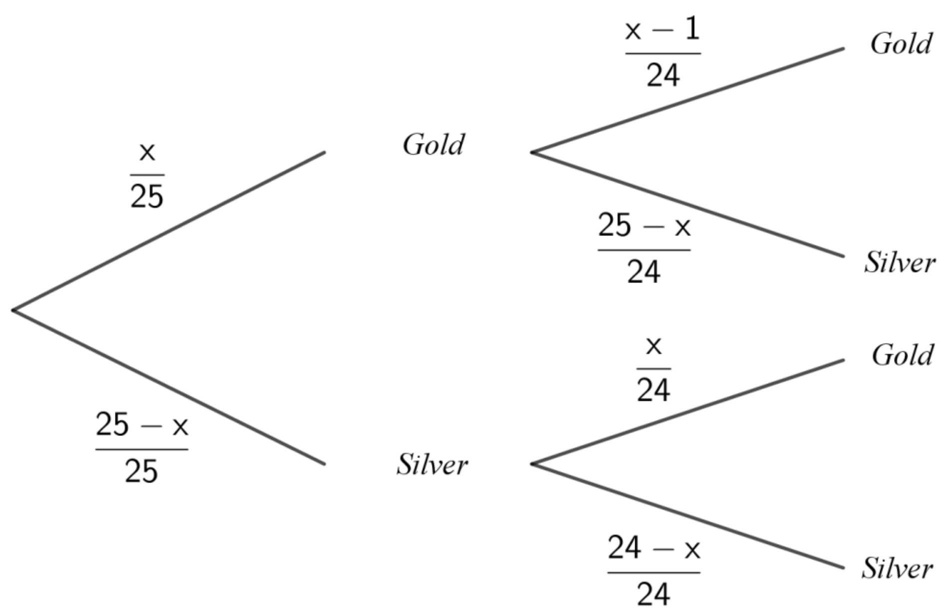
a) Show that the probability of selecting two gold tickets is $\frac{x^2-x}{600}$

b) Find the probability of selecting two tickets of the same colour.

c) The probability of selecting two tickets of the same colour is twice the probability of selecting two tickets of a different colour. Find how many gold tickets there are.

If x is the number of Gold tickets, then are $25-x$ Silver tickets.

It helps to draw a tree diagram to represent this situation:



a)

$$P(G\ AND\ G) = \frac{x}{25}\cdot\frac{x-1}{24} = \frac{x^2-x}{600}$$

b)

$$P(S\ AND\ S) = \frac{25-x}{25}\cdot\frac{24-x}{24} = \frac{600-25x-24x+x^2}{600}$$

$$= \frac{600 - 49x + x^2}{600}$$

$$P(\text{same colour}) = P(G \text{ AND } G) + P(S \text{ AND } S) = \frac{x^2 - x}{600} + \frac{600 - 49x + x^2}{600}$$

$$= \mathbf{\frac{2x^2 - 50x + 600}{600}}$$

c) If the probability of selecting two tickets of the same colour is ***twice*** the probability of selecting two tickets of a different colour

...then

the probability of selecting two tickets of a different colour $= \frac{1}{3}$

the probability of selecting two tickets of the same colour $= \frac{2}{3}$

$$\frac{2x^2 - 50x + 600}{600} = \frac{2}{3}$$

$$2x^2 - 50x + 600 = 400$$

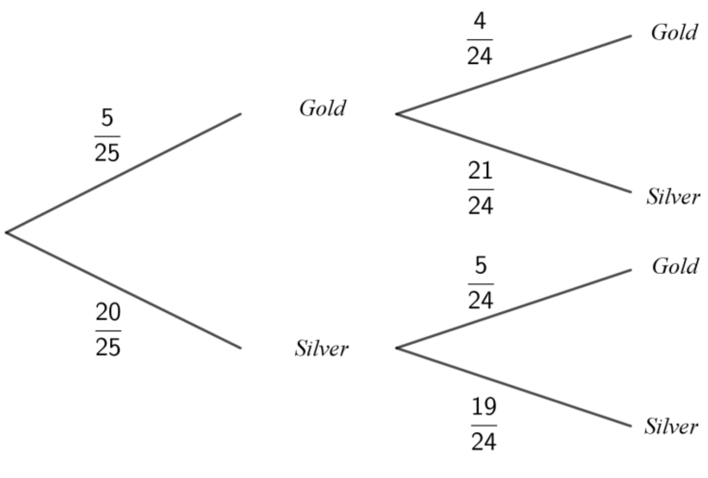
$$2x^2 - 50x + 200 = 0$$

$$x^2 - 25x + 100 = 0$$

$$(x - 20)(x - 5) = 0$$

$$\mathbf{x = 20 \text{ or } x = 5}$$

Let's just check that



$$P(G \text{ AND } G) + P(S \text{ AND } S) = \frac{5}{25} \cdot \frac{4}{24} + \frac{20}{25} \cdot \frac{19}{24} = \frac{400}{600} = \frac{2}{3}$$