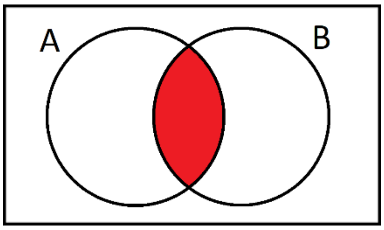
Probability Calculations

Venn Diagrams

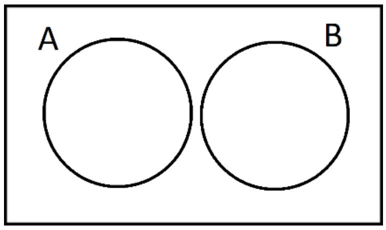
Having a good understanding of Venn diagrams can be very helpful for thinking about probability problems:

The Intersection: A ***AND*** B , $(A \cap B)$

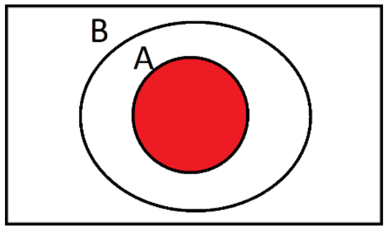
Two overlapping sets



2 disjoint sets

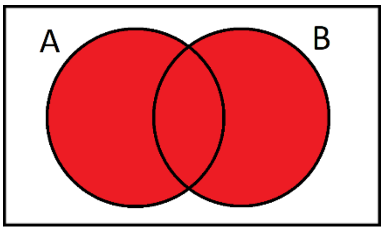


A is a subset of B

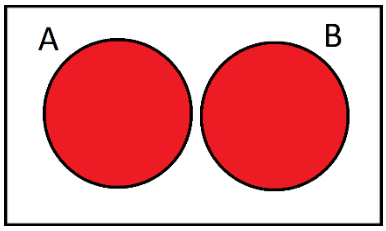


The Union: A ***OR*** B , $(A \cup B)$

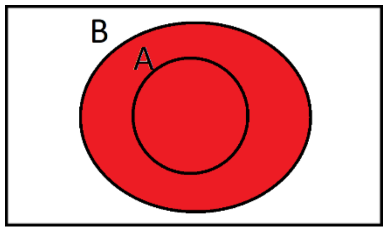
Two overlapping sets



2 disjoint sets

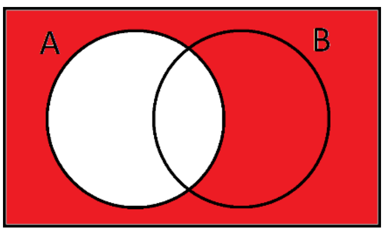


A is a subset of B

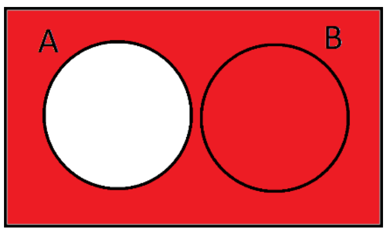


The Complement: ***NOT*** A , A'

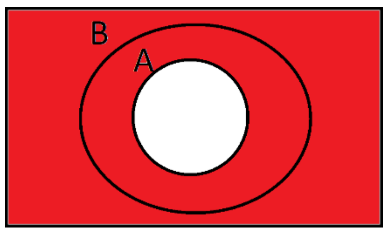
Two overlapping sets



2 disjoint sets



A is a subset of B



Probability Rule

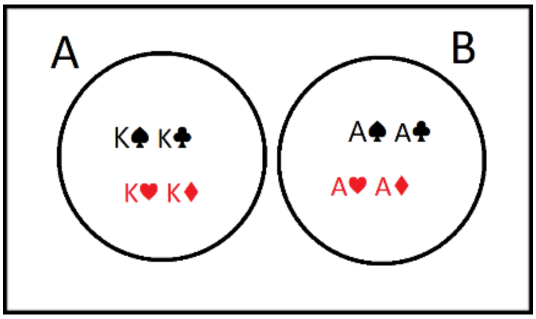
$$P(A \textbf{ OR } B) = P(A) + P(B) - P(A \textbf{ AND } B)$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Mutually Exclusive Events $P(A \cup B) = P(A) + P(B)$

Two events are mutually exclusive, then they cannot happen at the same time

For example, 'selecting a King', 'selecting an Ace' are mutually exclusive (a card cannot be a King and an Ace at the same time):



A = selecting a king

B = selecting an ace

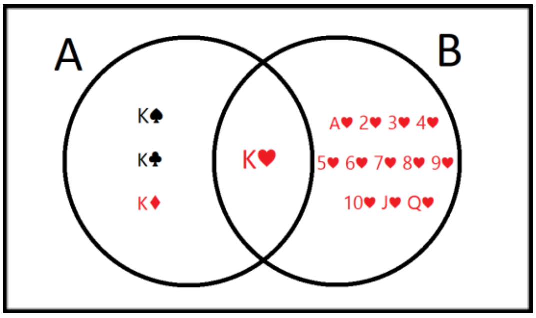
There is no intersection, $P(A \cap B) = 0$

So, the probability of $A \textbf{ OR } B$ becomes:

$P(A \cup B) = P(A) + P(B)$

If two events are **NOT mutually exclusive**, then they can happen at the same time

For example, 'selecting a King', 'selecting a Heart' are not mutually exclusive (a card can be both a King and a Heart):



A = selecting a king

B = selecting a heart

There is an intersection, the king of hearts!

If two events are NOT mutually exclusive that does not mean that they are independent

Independent Events $P(B|A) = P(B)$, $P(A \cap B) = P(A) \times P(B)$

Two events, A and B, are independent if the fact that A occurs **does not affect** the probability of B occurring. The probability of B happening given that A has happened is (still) the probability of B happening:

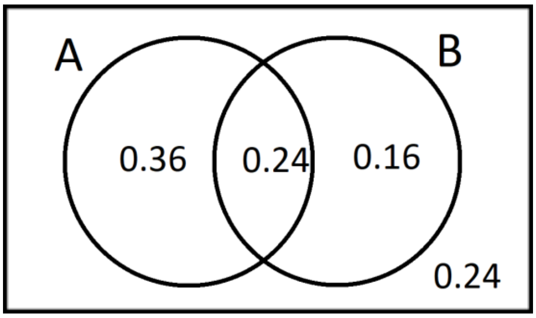
$P(B|A) = P(B)$

For example, 'rolling a 6 on a dice', 'rolling a 6 on a dice a second time' are independent (the dice does not have a memory!)

We can work out the probability of $A\ AND\ B$ happening by multiplying the probabilities

$P(A\ AND\ B) = P(A) \times P(B)$

$P(A \cap B) = P(A) \times P(B)$



$P(A) = 0.6$
$P(\text{B}) = 0.4$
$P(A \cap B) = 0.24$

In this case, A and B are independent since

$$P(A \cap B) = P(A) \times P(B)$$
$$0.24 = 0.6 \times 0.4$$

Dependent Events $P(B|A) \neq P(B)$

Two events, A and B, are dependent if the fact that A occurs **does affect** the probability of B occurring.

$P(B|A) \neq P(B)$

For example, 'drawing an Ace from a pack of cards', then **without replacing** it, 'drawing a second Ace' are dependent (the probability of drawing second time round is affected by what was drawn first.