$1 + \cos x + \cos^2 x + \cos 3x + \ldots = 2 + \sqrt{2}$

Find ***x*** given that $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$

$1 + \cos x + \cos^2 x + \cos 3x + \ldots$

…is an infinite geometric series

$U_1 = 1$

$r = cosx$

Sum to infinity $= \frac{U_1}{1-r}$

$$\frac{1}{1 - cosx} = 2 + \sqrt{2}$$

Rearrange the equation to make $cosx$ the subject

$$1 = (2 + \sqrt{2})(1 - cosx)$$

$$1 = 2 - 2cosx + \sqrt{2} - \sqrt{2}cosx$$

$$2cosx - \sqrt{2}cosx = 1 + \sqrt{2}$$

Factorise $(2 + \sqrt{2})cosx = 1 + \sqrt{2}$

$$cosx = \frac{1 + \sqrt{2}}{2 + \sqrt{2}}$$

We must solve this equation without a calculator so we should be able to simplify.

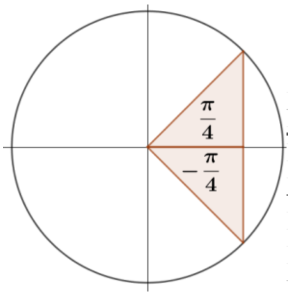
Rationalise the denominator

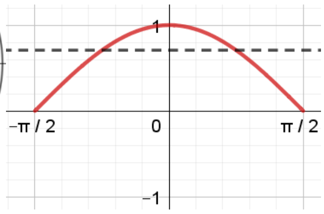
$$cosx = \frac{(1 + \sqrt{2})(2 - \sqrt{2})}{(2 + \sqrt{2})(2 - \sqrt{2})}$$

$$cosx = \frac{2 - \sqrt{2} + 2\sqrt{2} - 2}{4 - 2\sqrt{2} + 2\sqrt{2} - 2}$$

$$cosx = \frac{\sqrt{2}}{2}$$

Solve for $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$





$$\boldsymbol{x = -\frac{\pi}{4}, \frac{\pi}{4}}$$