The three planes Π_1, Π_2 and Π_3 meet at straight line
Π_1: $2x + y + 3z = a$
Π_2: $x - 2y + 2z = -9$
Π_3: $3x + 4y + 4z = -1$

a) Find $\boldsymbol{a}$
b) Find the equation of the straight line in the form $\boldsymbol{r} = \boldsymbol{a} + \lambda\boldsymbol{b}$ where the components of **b** are integers.

a)

$$2x + y + 3z = a \qquad A$$
$$x - 2y + 2z = -9 \qquad B$$
$$3x + 4y + 4z = -1 \qquad C$$

Eliminate x

$B \times 2$ $\quad 2x - 4y + 4z = -18 \qquad B \times 2$
$\quad 2x + y + 3z = a \qquad A$

$A - B \times 2$ $\quad 5y - z = a + 18$

$\quad 3x - 6y + 6z = -27 \qquad B \times 3$
$\quad 3x + 4y + 4z = -1 \qquad C$
$C - B \times 3$ $\quad 10y - 2z = 26$

Equate the coefficients of y and z
$(A - B \times 2) \times 2$ $\quad 10y - 2z = 2a + 36$
$\quad 10y - 2z = 26 \qquad C - B \times 3$

Given that the system can be solved

$$2a + 36 = 26$$
$$2a = -10$$
$$\boldsymbol{a = -5}$$

b)

Find y in terms of z
From $C - B \times 3$

$$10y - 2z = 26 \qquad C - B \times 3$$
$$10y = 2z + 26$$
$$y = \frac{z + 13}{5}$$

Find x in terms of z

$$2x + y + 3z = -5 \qquad A$$
$$2x + \frac{z + 13}{5} + 3z = -5$$
$$10x + z + 13 + 15z = -25$$
$$10x = -16z - 38$$
$$x = \frac{-8z - 19}{5}$$

Write equation of line

$$x = \frac{-8z - 19}{5}$$
$$y = \frac{z + 13}{5}$$
$$z = z$$

Let $z = \lambda$

$$x = \frac{-8\lambda - 19}{5}$$
$$y = \frac{\lambda + 13}{5}$$
$$z = \lambda$$

Write in vector form

$$\boldsymbol{r} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} \frac{-8\lambda - 19}{5} \\ \frac{\lambda + 13}{5} \\ \lambda \end{pmatrix}$$

$$\boldsymbol{r} = \begin{pmatrix} -\frac{19}{5} \\ \frac{13}{5} \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} \frac{1}{5} \\ \frac{1}{5} \\ 1 \end{pmatrix}$$

$\begin{pmatrix} \frac{1}{5} \\ \frac{1}{5} \\ 1 \end{pmatrix}$ is parallel to $\begin{pmatrix} 1 \\ 1 \\ 5 \end{pmatrix}$

$$\boldsymbol{r} = \begin{pmatrix} -\frac{19}{5} \\ \frac{13}{5} \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 1 \\ 5 \end{pmatrix}$$