

Infinite Solution - Finding Equation of a Line

Show that the following system of equations has infinite solutions and find the general solution of this system

$$\begin{aligned} x + 3y - 2z &= 7 & \textcircled{1} \\ 2x - 2y + z &= 3 & \textcircled{2} \\ 3x + y - z &= 10 & \textcircled{3} \end{aligned}$$

$$\textcircled{2} + \textcircled{3}$$

$$5x - y = 13$$

$$2 \times \textcircled{2}$$

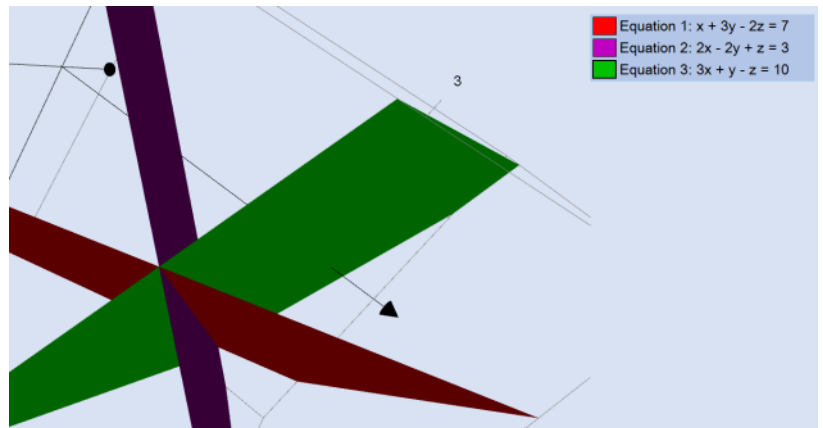
$$4x - 4y + 2z = 6 \quad \textcircled{A}$$

$$\textcircled{1}$$

$$x + 3y - 2z = 7 \quad \textcircled{B}$$

$$\textcircled{A} + \textcircled{B}$$

$$5x - y = 13$$



Infinite solutions

Finding the Equation of the Line

Method

1. Eliminate z, write y in terms of x
2. Eliminate y, write z in terms of x
3. Write equation of line in parametric form

1. Eliminate z, write y in terms of x

...we have already done this

$$y = 5x - 13$$

2. Eliminate y, write z in terms of x

$$\textcircled{1} \times 2$$

$$2x + 6y - 4z = 14 \quad \textcircled{A}$$

$$\textcircled{2} \times 3$$

$$6x - 6y + 3z = 9 \quad \textcircled{B}$$

$$\textcircled{A} + \textcircled{B}$$

$$8x - z = 23$$

$$8x - 23 = z$$

3. Write equation of line in parametric form

$$\begin{aligned}x &= x \\y &= 5x - 13 \\z &= 8x - 23\end{aligned}$$

$$\begin{aligned}x &= \lambda \\y &= 5\lambda - 13 \\z &= 8\lambda - 23\end{aligned}$$

Parametric form

Convert into other forms if necessary:

Vector form $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ -13 \\ -23 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 5 \\ 8 \end{pmatrix}$

Cartesian form

$$\begin{aligned}x &= \lambda \\ \frac{y + 13}{5} &= \lambda \\ \frac{z + 23}{8} &= \lambda\end{aligned} \qquad x = \frac{y + 13}{5} = \frac{z + 23}{8}$$