Intersection of Planes - Problem Solving

Find the value(s) of *k* for which the system of equations below has

a) Infinitely many solutions
b) A unique solution

$$2x - y + z = 0$$
$$3x + 2y - z = 8$$
$$x + ky + 3z = -k^2 + 8$$

When we eliminate one variable…

- Unique Solution - it works, we can solve it!
- Zero Solution - 2 inconsistent equations
- Infinite solutions - 2 identical equations

$$2x - y + z = 0 \quad (1)$$
$$3x + 2y - z = 8 \quad (2)$$
$$x + ky + 3z = -k^2 + 8 \quad (3)$$

(1)+(2)

$$5x + y = 8$$

3×(2)

$$9x + 6y - 3z = 24 \quad (A)$$

(3) $x + ky + 3z = -k^2 + 8$ (B)

(A)+(B)

$$10x + 6y + ky = 24 - k^2 + 8$$
$$10x + (6+k)y = 32 - k^2$$

$5x + y = 8$ (C)
$10x + (6+k)y = 32 - k^2$ (D)
2×(C) $10x + 2y = 16$

$6+k = 2$
$k = -4$

$32 - k^2 = 16$
$16 = k^2$
$k = \pm 4$

a) Infinite solutions when k = -4
b) Unique solution when $k \neq -4$