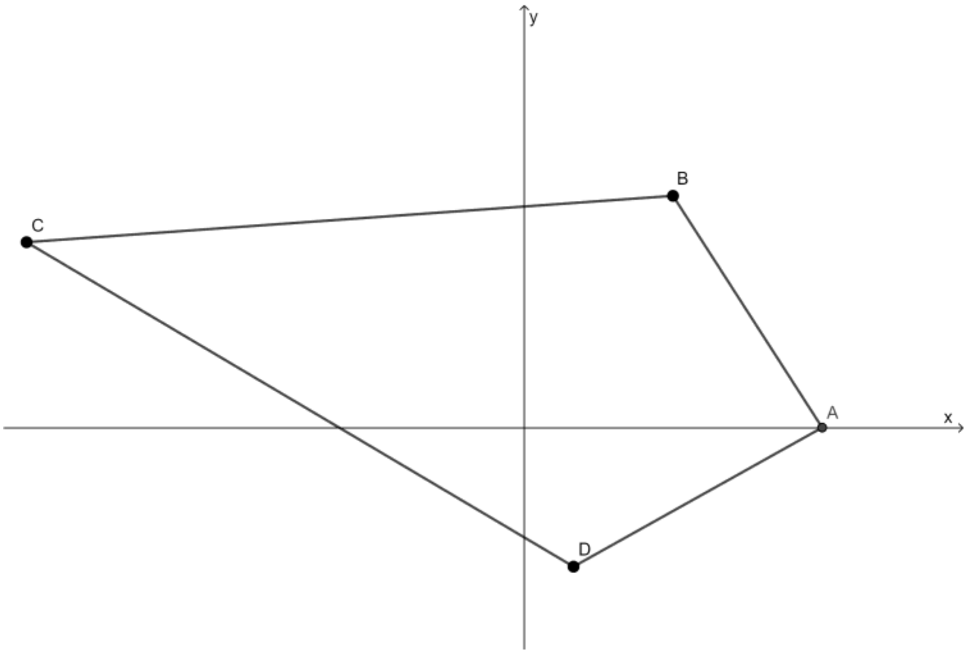
The diagram shows quadrilateral ABCD with vertices A(6,0) , B(3,5) , C(-10,4) and D(1,-3)



a) Find $\overrightarrow{AC}$
b) Show that $\overrightarrow{BD}$ is perpendicular to $\overrightarrow{AC}$
c) Write down the equation of the line (AC) in the form $\boldsymbol{r} = \boldsymbol{a} + t\boldsymbol{b}$.
d) Write down the equation of the line (BD)
e) The lines (AC) and (BD) intersect at E. Find the coordinates of E

a)

$$\overrightarrow{AC} = \overrightarrow{OC} - \overrightarrow{OA}$$
$$= \begin{pmatrix} -10 \\ 4 \end{pmatrix} - \begin{pmatrix} 6 \\ 0 \end{pmatrix}$$
$$= \begin{pmatrix} -16 \\ 4 \end{pmatrix}$$

b)

$$\overrightarrow{BD} = \begin{pmatrix} 1 \\ -3 \end{pmatrix} - \begin{pmatrix} 3 \\ 5 \end{pmatrix}$$
$$= \begin{pmatrix} -2 \\ -8 \end{pmatrix}$$

$$\overrightarrow{AC} \cdot \overrightarrow{BD} = \begin{pmatrix} -16 \\ 4 \end{pmatrix} \cdot \begin{pmatrix} -2 \\ -8 \end{pmatrix}$$
$$= 32 - 32$$
$$= 0$$

Since $\overrightarrow{AC} \cdot \overrightarrow{BD} = 0$ Then $\overrightarrow{AC}$ is perpendicular to $\overrightarrow{BD}$

c)

(AC) $\boldsymbol{r} = \begin{pmatrix} 6 \\ 0 \end{pmatrix} + s\begin{pmatrix} -16 \\ 4 \end{pmatrix}$

d)

(BD) $\boldsymbol{r} = \begin{pmatrix} 3 \\ 5 \end{pmatrix} + t\begin{pmatrix} -2 \\ -8 \end{pmatrix}$

e)

Find intersection of

$\boldsymbol{r} = \begin{pmatrix} 6 \\ 0 \end{pmatrix} + s\begin{pmatrix} -16 \\ 4 \end{pmatrix}$

and

$\boldsymbol{r} = \begin{pmatrix} 3 \\ 5 \end{pmatrix} + t\begin{pmatrix} -2 \\ -8 \end{pmatrix}$

$$6 - 16s = 3 - 2t$$

$$0 + 4s = 5 - 8t$$

$$-16s + 2t = -3$$

$$(4s + 8t = 5) \times 4$$

$$16s + 32t = 20$$

$$-16s + 2t = -3$$

$$34t = 17$$

$$t = 0.5$$

Substitute $t = 0.5$ in

$\boldsymbol{r} = \begin{pmatrix} 3 \\ 5 \end{pmatrix} + t\begin{pmatrix} -2 \\ -8 \end{pmatrix}$

$$\boldsymbol{r} = \begin{pmatrix} 3 \\ 5 \end{pmatrix} + 0.5\begin{pmatrix} -2 \\ -8 \end{pmatrix}$$

$$\boldsymbol{r} = \begin{pmatrix} 3 \\ 5 \end{pmatrix} + \begin{pmatrix} -1 \\ -4 \end{pmatrix}$$

$$\boldsymbol{r} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

$$E(2,1)$$