Two boats A and B, move so that a time t hours, their positions, in kilometres, are given by

$$r_A = \begin{pmatrix} -2 \\ -12 \end{pmatrix} + t \begin{pmatrix} 2 \\ -4 \end{pmatrix}$$

$$r_B = \begin{pmatrix} 11 \\ -11 \end{pmatrix} + t \begin{pmatrix} -2 \\ 3 \end{pmatrix}$$

a) Find the position where the two boats cross
b) Show that the boats do not collide

The key to answering this question correctly is to make the assumption that they will have the same position at some time, but that the time is not the same for the two boats.
Use t_A and t_B

Position of boat A: $\quad r_A = \begin{pmatrix} -2 \\ 12 \end{pmatrix} + t_A \begin{pmatrix} 2 \\ -4 \end{pmatrix}$

Position of boat B: $\quad r_B = \begin{pmatrix} 11 \\ -11 \end{pmatrix} + t_B \begin{pmatrix} -2 \\ 3 \end{pmatrix}$

$$-2 + 2t_A = 11 - 2t_B$$
$$12 - 4t_A = -11 + 3t_B$$

$$(2t_A + 2t_B = 13) \times 2$$
$$4t_A + 3t_B = 23$$
$$4t_A + 4t_B = 26$$

Eliminate t_A

$$t_B = 3$$

Find t_B $\quad 4t_A + 4(3) = 26$
$$4t_A = 14$$
$$t_A = 3.5$$

Find position of boat A at $t = 3.5$

$$r_A = \begin{pmatrix} -2 \\ 12 \end{pmatrix} + 3.5 \begin{pmatrix} 2 \\ -4 \end{pmatrix}$$
$$r_A = \begin{pmatrix} 5 \\ -2 \end{pmatrix}$$

Check the position of boat B at $t = 3$

$$r_B = \begin{pmatrix} 11 \\ -11 \end{pmatrix} + 3 \begin{pmatrix} -2 \\ 3 \end{pmatrix}$$
$$r_B = \begin{pmatrix} 5 \\ -2 \end{pmatrix}$$

Hence the boats have the same position at $(5, -2)$

A gets to $(5, -2)$ at t = 3.5

B gets to $(5, -2)$ at t = 3

Hence they do not arrive at $(5, -2)$ at the same time

The boats do not collide