The line L_1 has equation $\boldsymbol{r} = \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} + s\begin{pmatrix} -1 \\ -2 \\ k \end{pmatrix}$.

The line L_2 has equation $\boldsymbol{r} = \begin{pmatrix} 2 \\ 1 \\ -3 \end{pmatrix} + t\begin{pmatrix} 2 \\ -3 \\ -1 \end{pmatrix}$.

a) The point A(3,1,-1) lies on the line L_1. Show that $k = 4$
b) Show that the lines L_1 and L_2 are perpendicular.
c) Show that the lines L_1 and L_2 do not intersect.
d) The point B lies on the line L_1. The point C has coordinates (2,1,-3). ABC forms an isosceles triangles with AC=BC. Find the coordinates of B.

a)

A(3,1,-1) lies on L_1

$$\begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} + s\begin{pmatrix} -1 \\ -2 \\ k \end{pmatrix} = \begin{pmatrix} 3 \\ 1 \\ -1 \end{pmatrix}$$

$$\begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} + (-1)\begin{pmatrix} -1 \\ -2 \\ k \end{pmatrix} = \begin{pmatrix} 3 \\ 1 \\ -1 \end{pmatrix}$$

$$3 - k = -1$$

$$4 = k$$

b)

Direction of L_1 $\begin{pmatrix} -1 \\ -2 \\ 4 \end{pmatrix}$

Direction of L_2 $\begin{pmatrix} 2 \\ -3 \\ -1 \end{pmatrix}$

Find scalar product $\begin{pmatrix} -1 \\ -2 \\ 4 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ -3 \\ -1 \end{pmatrix} = -2 + 6 - 4$

Scalar product = 0

Lines are perpendicular

c)

If lines intersect $\begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} + s\begin{pmatrix} -1 \\ -2 \\ 4 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ -3 \end{pmatrix} + t\begin{pmatrix} 2 \\ -3 \\ -1 \end{pmatrix}$

$$2 - s = 2 + 2t$$

$$-1 - 2s = 1 - 3t$$

$(-\ s - 2t = 0) \times 2$

$-2s + 3t = 2$

$-\ 2s - 4t = 0$

$7t = 2$

$t = \frac{2}{7}$

$-\ s - 2\left(\frac{2}{7}\right) = 0$

$s = -\frac{4}{7}$

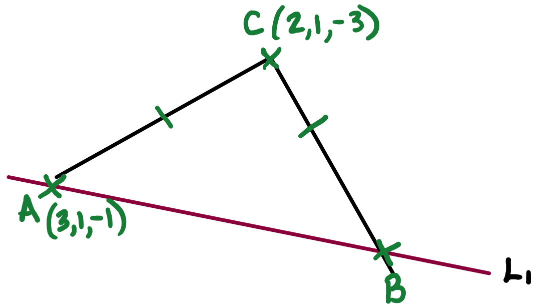
$$\boldsymbol{r} = \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} - \frac{4}{7}\begin{pmatrix} -1 \\ -2 \\ 4 \end{pmatrix} = \begin{pmatrix} \frac{18}{7} \\ \frac{1}{7} \\ \frac{5}{7} \end{pmatrix}$$

$$\boldsymbol{r} = \begin{pmatrix} 2 \\ 1 \\ -3 \end{pmatrix} + \frac{2}{7}\begin{pmatrix} 2 \\ -3 \\ -1 \end{pmatrix} = \begin{pmatrix} \frac{18}{7} \\ \frac{1}{7} \\ -\frac{23}{7} \end{pmatrix}$$

Since $\frac{5}{7} \neq -\frac{23}{7}$

The lines do not intersect.

d)



$|AC| = |BC|$

$|AC|^2 = (2-3)^2 + (1-1)^2 + (-3+1)^2$

$|AC|^2 = 1 + 0 + 4$

$|AC| = \sqrt{5}$

$$\boldsymbol{r} = \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} + s\begin{pmatrix} -1 \\ -2 \\ 4 \end{pmatrix}$$

$$\overrightarrow{OB} = \begin{pmatrix} 2-s \\ -1-2s \\ 3+4s \end{pmatrix}$$

$$\overrightarrow{BC} = \begin{pmatrix} 2 \\ 1 \\ -3 \end{pmatrix} - \begin{pmatrix} 2-s \\ -1-2s \\ 3+4s \end{pmatrix} = \begin{pmatrix} s \\ 2+2s \\ -6-4s \end{pmatrix}$$

$$|AC| = |BC|$$

$$s^2 + (2+2s)^2 + (-6-4s)^2 = 5$$

$$s^2 + 4 + 8s + 4s^2 + 36 + 48s + 16s^2 = 5$$

$$21s^2 + 56s + 35 = 0$$

$$3s^2 + 8s + 5 = 0$$

$$(3s+5)(s+1) = 0$$

$$s = -\frac{5}{3}, s = -1$$

When $s = -1$ $$\overrightarrow{OB} = \begin{pmatrix} 2+1 \\ -1+2 \\ 3-4 \end{pmatrix} = \begin{pmatrix} 3 \\ 1 \\ -1 \end{pmatrix}, \textit{this is point } A$$

When $s = -\frac{5}{3}$

$$\overrightarrow{OB} = \begin{pmatrix} 2+\frac{5}{3} \\ -1+\frac{10}{3} \\ 3-\frac{20}{3} \end{pmatrix} = \begin{pmatrix} \frac{11}{3} \\ \frac{7}{3} \\ -\frac{11}{3} \end{pmatrix}$$

$$B\left(\frac{11}{3}, \frac{7}{3}, -\frac{11}{3}\right)$$