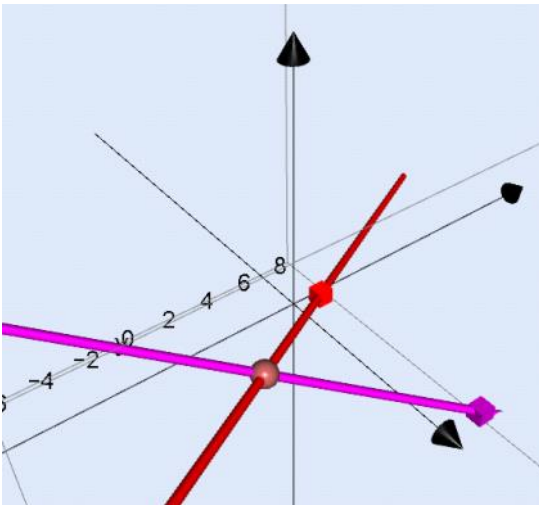


Intersection of 2 lines

Find the coordinates of the point of intersection of the following two lines

$$r_1 = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix} \quad r_2 = \begin{pmatrix} 5 \\ 1 \\ 0 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$$



$$r_1 = \begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix} \quad r_2 = \begin{pmatrix} x_2 \\ y_2 \\ z_2 \end{pmatrix} = \begin{pmatrix} 5 \\ 1 \\ 0 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$$

For two lines to intersect

$$\begin{aligned} x_1 &= x_2 \\ y_1 &= y_2 \\ z_1 &= z_2 \end{aligned}$$

$$\begin{aligned} x_1 &= 1 + 2\lambda & x_2 &= 5 + \mu \\ y_1 &= 0 - 3\lambda & y_2 &= 1 + 2\mu \\ z_1 &= 1 + \lambda & z_2 &= 0 - \mu \end{aligned}$$

$$\begin{aligned} 1 + 2\lambda &= 5 + \mu \\ -3\lambda &= 1 + 2\mu \\ 1 + \lambda &= 0 - \mu \end{aligned}$$

$$\begin{aligned} 2\lambda - \mu &= 4 & A \\ -3\lambda - 2\mu &= 1 & B \\ \lambda + \mu &= -1 & C \end{aligned}$$

$$\begin{aligned} 2\lambda - \mu &= 4 & A \\ \lambda + \mu &= -1 & C \end{aligned}$$

$$\begin{aligned} A+C & \quad 3\lambda = 3 \\ & \quad \lambda = 1 \\ \text{Sub in C} & \quad 1 + \mu = -1 \\ & \quad \mu = -2 \end{aligned}$$

Check that these values hold true for B

$$-3\lambda - 2\mu = -3 - 2(-2) = 1$$

Find coordinates of point of intersection

$$\begin{aligned} x_1 &= 1 + 2\lambda & x_2 &= 5 + \mu \\ y_1 &= 0 - 3\lambda & y_2 &= 1 + 2\mu \\ z_1 &= 1 + \lambda & z_2 &= 0 - \mu \end{aligned}$$

Hence the two lines intersect at $(3, -3, 2)$