During an air show, two planes, A and B, perform a manoeuvre in which their paths cross in a *near miss*. The two planes are flying at the same altitude.

$$\mathbf{r}_A = \begin{pmatrix} 150 \\ 320 \end{pmatrix} + t\begin{pmatrix} 200 \\ 300 \end{pmatrix}$$

$$\mathbf{r}_B = \begin{pmatrix} 875 \\ 110 \end{pmatrix} + t\begin{pmatrix} -100 \\ 400 \end{pmatrix}$$

t = time in seconds. Distances are given in metres.

a) Show that the two planes cross paths, but the planes do not collide

b) Find the distance between the planes when t = 0.

c) Show that the distance **d** between A and B at any time t can be given by the expression

d = $\sqrt{100000t^2 - 477000t + 569725}$

d) To the nearest metre, find the closest distance that the two planes get to one another.

a) $V_B = \begin{pmatrix} -100 \\ 400 \end{pmatrix}$ $V_A = \begin{pmatrix} 200 \\ 300 \end{pmatrix}$

since $\begin{pmatrix} -100 \\ 400 \end{pmatrix} \neq k\begin{pmatrix} 200 \\ 300 \end{pmatrix}$ the direction of the planes is not parallel

Therefore, their paths will cross.

$\underline{r}_A = \begin{pmatrix} 150 \\ 320 \end{pmatrix} + t\begin{pmatrix} 200 \\ 300 \end{pmatrix}$

$\underline{r}_B = \begin{pmatrix} 875 \\ 110 \end{pmatrix} + t\begin{pmatrix} -100 \\ 400 \end{pmatrix}$

$$150 + 200t = 875 - 100t$$
$$300t = 725$$
$$t \approx 2.42$$

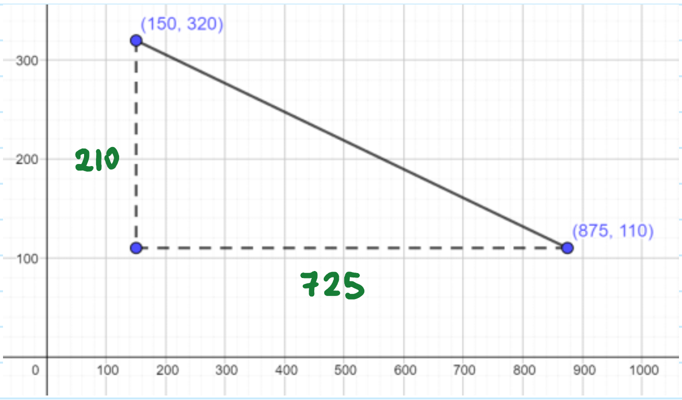
$$320 + 300t = 110 + 400t$$
$$210 = 100t$$
$$t = 2.1$$

The x positions of the planes are the same when $t \approx 2.42$

The y positions of the planes are the same when $t = 2.1$

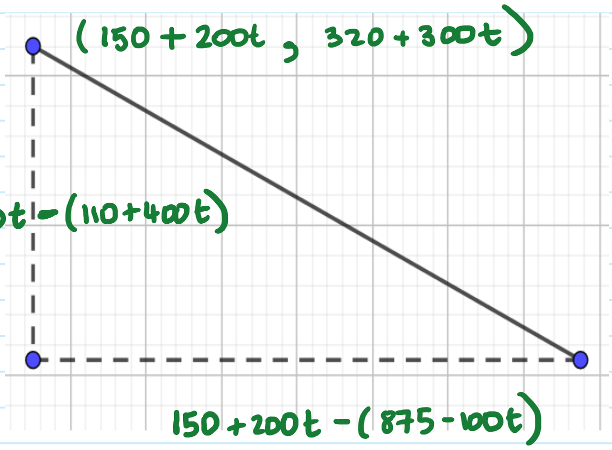
Hence the planes do <u>not</u> collide.

b) When $t = 0$ $\underline{r}_A = \begin{pmatrix} 150 \\ 320 \end{pmatrix}$ $r_B = \begin{pmatrix} 875 \\ 110 \end{pmatrix}$



$$\text{distance} = \sqrt{725^2 + 210^2}$$
$$\approx 755\text{m}$$

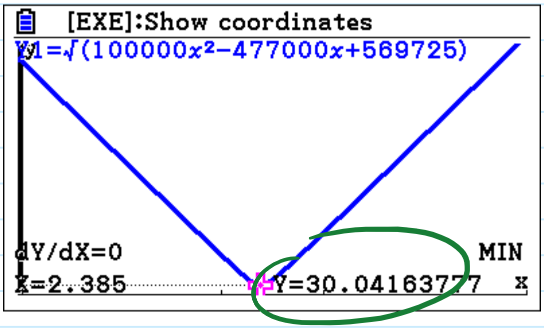
c) $(150 + 200t\ ,\ 320 + 300t)$



$(875 - 100t\ ,\ 110 + 400t)$

$$d^2 = (150 + 200t - (875 - 100t))^2 + (320 + 300t - (110 + 400t))^2$$
$$d^2 = (-725 + 300t)^2 + (210 - 100t)^2$$
$$d = \sqrt{100\,000t^2 - 477\,000t + 569\,725}$$

d)



Closest distance $\approx$ 30m