Distances in this question are given in metres

Two brothers, Orville and Wilbur are testing their model airplanes. The position of Orville's airplane ***t*** seconds after taking off from ground level is given by

$$\mathbf{r} = \begin{pmatrix} 12 \\ -19 \\ 0 \end{pmatrix} + t \begin{pmatrix} -4 \\ 4 \\ 3 \end{pmatrix}$$

a) Find the height of the plane after 4 seconds.

b) Wilbur's airplane takes off **after** Wilbur's airplane ***s*** seconds after taking off fis given by

$$\mathbf{r} = \begin{pmatrix} -26 \\ 25 \\ 0 \end{pmatrix} + s \begin{pmatrix} 2 \\ -4 \\ 8 \end{pmatrix}$$

Find the angle between the two paths.

c) The two airplanes collide at (-20,13,24). How long after Orville's airplane takes off does Wilbur's airplane take off?

d) Find the speed of the two airplanes at the moment of the collision.

a) height is given by z position

$$\mathbf{r} = \begin{pmatrix} 12 \\ -19 \\ 0 \end{pmatrix} + t \begin{pmatrix} -4 \\ 4 \\ 3 \end{pmatrix}$$

When $t = 0$ height $= 0 + 4 \times 3$
$= 12m$

b) direction of Orville's airplane $= \begin{pmatrix} -4 \\ 4 \\ 3 \end{pmatrix}$

direction of Wilbur's airplane $= \begin{pmatrix} 2 \\ -4 \\ 8 \end{pmatrix}$

Find scalar product $\begin{pmatrix} -4 \\ 4 \\ 3 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ -4 \\ 8 \end{pmatrix} = -8 - 16 + 24 = 0$

Since the scalar product $= 0$, the vectors are perpendicular

Angle $= 90°$

c) Find time to $(-20, 13, 24)$

$$\begin{pmatrix} 12 \\ -19 \\ 0 \end{pmatrix} + t \begin{pmatrix} -4 \\ 4 \\ 3 \end{pmatrix} = \begin{pmatrix} -20 \\ 13 \\ 24 \end{pmatrix} \qquad 0 + 3t = 24, \quad t = 8$$

$$\begin{pmatrix} -26 \\ 25 \\ 0 \end{pmatrix} + s \begin{pmatrix} 2 \\ -4 \\ 8 \end{pmatrix} = \begin{pmatrix} -20 \\ 13 \\ 24 \end{pmatrix} \qquad 0 + 8s = 24, \quad s = 3$$

Wilbur's airplane takes off 5 seconds after Orville's.

d) Velocity of Orville's airplane = $V_o = \begin{pmatrix} -4 \\ 4 \\ 3 \end{pmatrix}$ ms^{-1}

Velocity of Wilbur's airplane = $V_w = \begin{pmatrix} 2 \\ -4 \\ 8 \end{pmatrix}$

Speed of Orville's airplane = $|V_o| = \sqrt{(-4)^2 + 4^2 + 3^2}$

$= \sqrt{41}$

$\approx 6.40 ms^{-1}$

Speed of Wilbur's airplane = $|V_w| = \sqrt{2^2 + (-4)^2 + 8^2}$

$= \sqrt{84}$

$\approx 9.17 ms^{-1}$