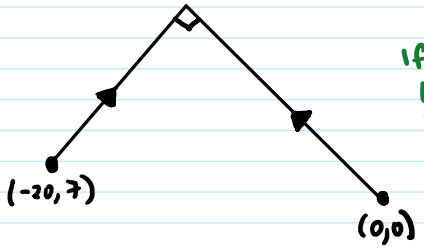
All distances in this question are in km.

An interceptor missile, M_1 is positioned at the origin. A missile, M_2 is launched from (-20,7) with velocity $\begin{pmatrix}3\\1\end{pmatrix}$ kms^{-1}. M_1 is capable of twice the speed of M_2. How many seconds later must the interceptor missile, M_1 be launched if it is to travel the **shortest possible distance**?

The position of M_2 can be given by $\underline{r}_2 = \begin{pmatrix}-20\\7\end{pmatrix} + t\begin{pmatrix}3\\1\end{pmatrix}$
where t is time in seconds after its launch



If M_1 is to travel shortest distance then its direction must be perpendicular to M_2

Speed of M_1 = 2 × speed of M_2

Velocity of $M_1 = \begin{pmatrix}-2\\6\end{pmatrix}$

Note that $\begin{pmatrix}3\\1\end{pmatrix}\cdot\begin{pmatrix}-2\\6\end{pmatrix} = 0$

If M_1 is launched a seconds after M_2, position of M_2 can be given by $\underline{r}_1 = (t-a)\begin{pmatrix}-2\\6\end{pmatrix}$, $t \geq a$

M_1 collides with M_2

$$\begin{pmatrix}-20\\7\end{pmatrix} + t\begin{pmatrix}3\\1\end{pmatrix} = (t-a)\begin{pmatrix}-2\\6\end{pmatrix}$$

Find a

Equate x positions: $-20 + 3t = -2(t-a) \Rightarrow -20 + 3t = -2t + 2a \Rightarrow 5t - 2a = 20$ (1)

Equate y positions: $7 + t = 6(t-a) \Rightarrow 7 + t = 6t - 6a \Rightarrow 5t - 6a = 7$ (2)

(1) − (2) $4a = 13$

$a = 3.25$ $\left(t = \frac{7 + 6(3.25)}{5} = 5.3\right)$

M_1 should be launched 3.25 seconds after M_2