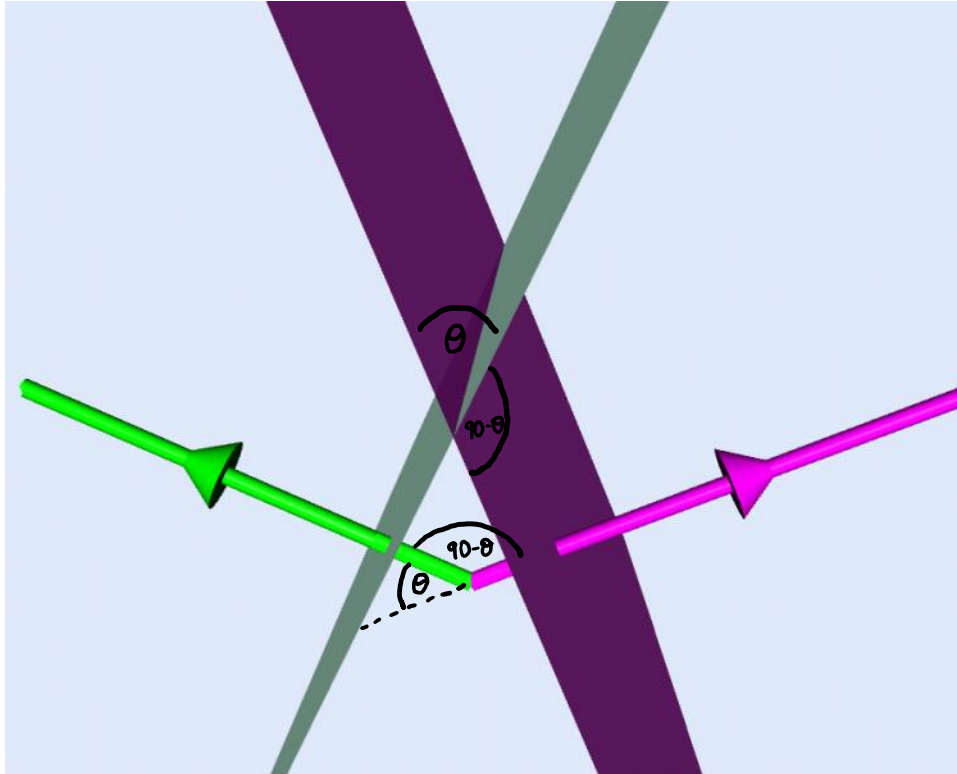


## Angle between 2 planes

Find the **acute** angle between the planes  $2x + 3y - 4z = 6$  and  $x - y + 2z = 2$

angle between two planes = angle between normals



There are two possible angles between 2 planes: an acute and an obtuse angle

$$2x + 3y - 4z = 6$$
$$\text{Normal} = \begin{pmatrix} 2 \\ 3 \\ -4 \end{pmatrix}$$

$$1x - 1y + 2z = 2$$
$$\text{Normal} = \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$$

Find angle between two direction vectors  $\begin{pmatrix} 2 \\ 3 \\ -4 \end{pmatrix}$  and  $\begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$

Angle between 2 vectors **a** and **b**

$$\cos\theta = \frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}||\mathbf{b}|}$$

$$\begin{pmatrix} 2 \\ 3 \\ -4 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix} = 2 \cdot 1 + 3 \cdot (-1) + (-4) \cdot 2 \\ = -9$$

$$\left| \begin{pmatrix} 2 \\ 3 \\ -4 \end{pmatrix} \right| = \sqrt{2^2 + 3^2 + (-4)^2} = \sqrt{29}$$

$$\left| \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix} \right| = \sqrt{1^2 + (-1)^2 + 2^2} = \sqrt{6}$$

$$\cos\theta = \frac{-9}{\sqrt{29}\sqrt{6}}$$

$$\theta \approx 133^\circ$$

Angle between planes  $\approx 47^\circ$  *or*  $133^\circ$

Acute angle between planes  $\approx 47^\circ$