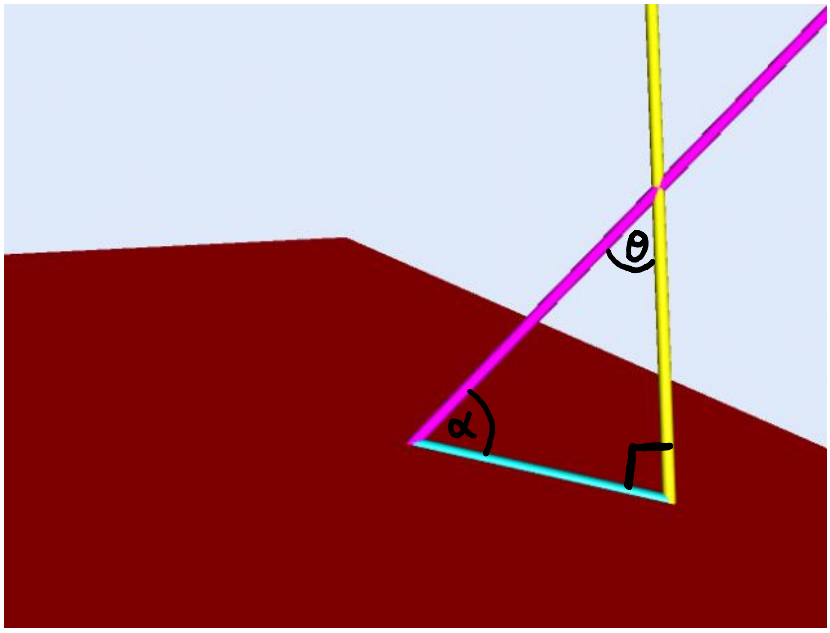
Angle between a line and plane

Find the acute angle between the line and plane

$$-x = \frac{y-5}{2} = 2z - 8 \qquad 3x - y + z = 8$$



Angle between line and plane = α
Angle between line and normal = θ
α = 90 - θ

Find the direction of the line

$$-x = \frac{y-5}{2} = 2z - 8 = \lambda$$

Find normal to the plane

$3x - 1y + 1z = 8$

Normal vector = $\begin{pmatrix} 3 \\ -1 \\ 1 \end{pmatrix}$

Write in parametric form

$$-x = \lambda \qquad \frac{y-5}{2} = \lambda \qquad 2z - 8 = \lambda$$

$$x = -\lambda \qquad y = 5 + 2\lambda \qquad z = 4 + \frac{1}{2}\lambda$$

Write in vector form

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 5 \\ 4 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 2 \\ 0.5 \end{pmatrix}$$

Direction of line = $\begin{pmatrix} -1 \\ 2 \\ 0.5 \end{pmatrix}$ *which is parallel to* $\begin{pmatrix} -2 \\ 4 \\ 1 \end{pmatrix}$

Find angle between two direction vectors

$\begin{pmatrix} -2 \\ 4 \\ 1 \end{pmatrix}$ *and* $\begin{pmatrix} 3 \\ -1 \\ 1 \end{pmatrix}$

Angle between 2 vectors **a** and **b**

$$\boldsymbol{cos\theta} = \frac{\boldsymbol{a} \cdot \boldsymbol{b}}{|\boldsymbol{a}||\boldsymbol{b}|}$$

$$\begin{pmatrix} -2 \\ 4 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ -1 \\ 1 \end{pmatrix} = (-2) \cdot 3 + 4 \cdot (-1) + 1 \cdot 1 = -9$$

$$\left| \begin{pmatrix} -2 \\ 4 \\ 1 \end{pmatrix} \right| = \sqrt{(-2)^2 + 4^2 + 1^2} = \sqrt{21}$$

$$\left| \begin{pmatrix} 3 \\ -1 \\ 1 \end{pmatrix} \right| = \sqrt{3^2 + (-1)^2 + 1^2} = \sqrt{11}$$

$$\boldsymbol{cos\theta} = \frac{-9}{\sqrt{21}\sqrt{11}}$$

$$\boldsymbol{\theta \approx 126° \; or \; 180° - 126° \approx 54°}$$

Acute angle between normal and line $\approx \mathbf{54°}$

Acute angle between plane and line $\approx \mathbf{90 - 54°}$

$\approx \mathbf{36°}$