

The coordinates of points A, B and C are given as (5,4,1), (5,1,-2) and (1,-1,2) respectively.

a) Find the equation of the plane that passes through A, B and C

b) Find the equation of the plane that is perpendicular to AB and passes through C

$$a) \vec{OA} = \begin{pmatrix} 5 \\ 4 \\ 1 \end{pmatrix} \quad \vec{OB} = \begin{pmatrix} 5 \\ 1 \\ -2 \end{pmatrix} \quad \vec{OC} = \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$$

$$\vec{AB} = \vec{OB} - \vec{OA} \quad \vec{AC} = \vec{OC} - \vec{OA}$$
$$= \begin{pmatrix} 0 \\ -3 \\ -3 \end{pmatrix} \quad = \begin{pmatrix} -4 \\ -5 \\ 1 \end{pmatrix}$$

$$\vec{AB} \times \vec{AC} = \begin{pmatrix} 0 \\ -3 \\ -3 \end{pmatrix} \times \begin{pmatrix} -4 \\ -5 \\ 1 \end{pmatrix} = \begin{pmatrix} -3 \cdot 1 - (-3) \cdot (-5) \\ -(0 \cdot 1 - (-3) \cdot (-4)) \\ 0 \cdot (-5) - (-3) \cdot (-4) \end{pmatrix}$$
$$= \begin{pmatrix} -18 \\ 12 \\ -12 \end{pmatrix}$$
$$= -6 \begin{pmatrix} 3 \\ -2 \\ 2 \end{pmatrix}$$

$$r \cdot n = a \cdot n$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} \cdot \begin{pmatrix} 3 \\ -2 \\ 2 \end{pmatrix} = \begin{pmatrix} 5 \\ 4 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ -2 \\ 2 \end{pmatrix}$$

$$3x + (-2)y + 2z = 5 \cdot 3 + 4 \cdot (-2) + 1 \cdot 2$$

$$3x - 2y + 2z = 9$$

$$b) \vec{AB} = \begin{pmatrix} 0 \\ -3 \\ -3 \end{pmatrix} = -3 \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$$

$$r \cdot n = a \cdot n$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$$

$$y + z = 1$$