HL

A plane has vector equation $\mathbf{r} = \begin{pmatrix}1\\2\\0\end{pmatrix} + \mu\begin{pmatrix}2\\1\\1\end{pmatrix} + \lambda\begin{pmatrix}3\\0\\-1\end{pmatrix}$

Show that the Cartesian equation of the plane is x - 5y + 3z + 9 = 0

a point in the plane

a direction vector in the plane

a direction vector in the plane

Find the normal to the plane

$$\begin{pmatrix}2\\1\\1\end{pmatrix} \times \begin{pmatrix}3\\0\\-1\end{pmatrix} = \begin{pmatrix}1\cdot(-1) - 1\cdot 0\\ -(2\cdot(-1) - 1\cdot 3)\\ 2\cdot 0 - 1\cdot 3\end{pmatrix}$$

$$= \begin{pmatrix}-1\\5\\-3\end{pmatrix}$$

$$\mathbf{r}\cdot\mathbf{n} = \mathbf{a}\cdot\mathbf{n}$$

$$\begin{pmatrix}x\\y\\z\end{pmatrix}\cdot\begin{pmatrix}-1\\5\\-3\end{pmatrix} = \begin{pmatrix}1\\2\\0\end{pmatrix}\cdot\begin{pmatrix}-1\\5\\-3\end{pmatrix}$$

$$(-1)\cdot x + 5\cdot y + (-3)\cdot z = 1\cdot(-1) + 2\cdot 5 + 0\cdot(-3)$$

$$-x + 5y - 3z = 9$$

$$x - 5y + 3z = -9$$

$$x - 5y + 3z + 9 = 0$$