Equation of Planes

The equation of a plane can be given in several forms. It is important that you understand all the different forms

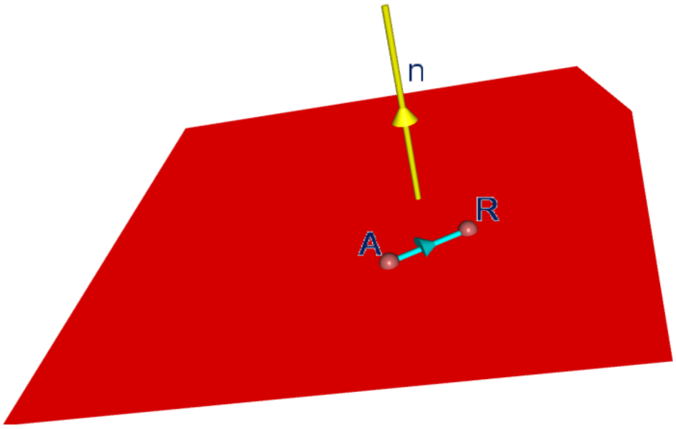
The ***vector*** and ***parametric form*** are almost identical

$\boldsymbol{r} = \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} + \mu \begin{pmatrix} 3 \\ 0 \\ -1 \end{pmatrix}$	***Vector Form***	(1, 2, 0) is a point that lies in the plane $\begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} \begin{pmatrix} 3 \\ 0 \\ -1 \end{pmatrix}$ are two direction vectors that lie in the plane
$x = 1 + 2\lambda + 3\mu$ $y = 2 + 1\lambda + 0\mu$ $z = 0 + 1\lambda + (-1)\mu$	***Parametric Form***	This is just the same as **r** can be written $\begin{pmatrix} x \\ y \\ z \end{pmatrix}$

To find the ***Normal form***, we need a vector perpendicular to the plane (the normal) and a point in the plane.

We often use the vector product (the formula is given in the formula booklet) to find the normal.

Therefore, we can easily convert from ***vector form***



$\begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} \times \begin{pmatrix} 3 \\ 0 \\ -1 \end{pmatrix} = \begin{pmatrix} -1 \\ 5 \\ -3 \end{pmatrix}$ = normal vector to the plane

$\boldsymbol{r} \cdot \boldsymbol{n} = \boldsymbol{a} \cdot \boldsymbol{n}$ $\begin{pmatrix} \boldsymbol{x} \\ \boldsymbol{y} \\ \boldsymbol{z} \end{pmatrix} \cdot \begin{pmatrix} \mathbf{-1} \\ \mathbf{5} \\ \mathbf{-3} \end{pmatrix} = \begin{pmatrix} \mathbf{1} \\ \mathbf{2} \\ \mathbf{0} \end{pmatrix} \cdot \begin{pmatrix} \mathbf{-1} \\ \mathbf{5} \\ \mathbf{-3} \end{pmatrix}$	Normal Form	Vector $\overrightarrow{AR}$ is perpendicular to **n** $(\boldsymbol{r} - \boldsymbol{a}) \cdot \boldsymbol{n} = \mathbf{0}$
$-x + 5y - 3z = -9$	Cartesian Form	Just work out the scalar products

You should notice that the ***normal*** and ***Cartesian forms*** are almost identical.

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