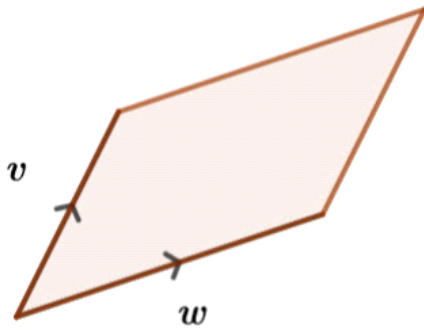


A parallelogram has two adjacent sides formed by the vectors $2i - j + 3k$ and $ai + 4k$. The area of the parallelogram is $\sqrt{26}$. Find the possible values of a .



$$\text{Area of parallelogram} = |\mathbf{v} \times \mathbf{w}|$$

$$\begin{aligned} \mathbf{v} &= 2i - j + 3k \\ \mathbf{w} &= ai + 4k \end{aligned}$$

$$\begin{aligned} \mathbf{v} \times \mathbf{w} &= \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} \times \begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix} = \begin{pmatrix} v_2 w_3 - v_3 w_2 \\ -(v_1 w_3 - v_3 w_1) \\ v_1 w_2 - v_2 w_1 \end{pmatrix} \\ &= \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} \times \begin{pmatrix} a \\ 0 \\ 4 \end{pmatrix} = \begin{pmatrix} (-1) \times 4 - 3 \times 0 \\ -(2 \times 4 - 3 \times a) \\ 2 \times 0 - (-1) \times a \end{pmatrix} \\ &= \begin{pmatrix} -4 \\ 3a - 8 \\ a \end{pmatrix} \end{aligned}$$

$$|\mathbf{v} \times \mathbf{w}| = \sqrt{(-4)^2 + (3a - 8)^2 + a^2} \quad |\mathbf{v} \times \mathbf{w}| = \sqrt{26}$$

$$\sqrt{(-4)^2 + (3a - 8)^2 + a^2} = \sqrt{26}$$

$$(-4)^2 + (3a - 8)^2 + a^2 = 26$$

$$16 + 9a^2 - 48a + 64 + a^2 = 26$$

$$10a^2 - 48a + 54 = 0$$

$$5a^2 - 24a + 27 = 0$$

$$(5a - 9)(a - 3) = 0$$

$$a = \frac{9}{5}, \quad a = 3$$

