Prove that $|\mathbf{v} \times \mathbf{w}|^2 = |\mathbf{v}|^2|\mathbf{w}|^2 - (\mathbf{v} \cdot \mathbf{w})^2$

Useful formula

$|\boldsymbol{v} \times \boldsymbol{w}| = |\boldsymbol{v}||\boldsymbol{w}|sin\theta$

$v \cdot w = |\boldsymbol{v}||\boldsymbol{w}|cos\theta$

$|\boldsymbol{v}|^2|\boldsymbol{w}|^2 - (\boldsymbol{v} \cdot \boldsymbol{w})^2$

$v \cdot w = |\boldsymbol{v}||\boldsymbol{w}|cos\theta$

$$|\boldsymbol{v}|^2|\boldsymbol{w}|^2 - (\boldsymbol{v} \cdot \boldsymbol{w})^2 = |\boldsymbol{v}|^2|\boldsymbol{w}|^2 - (|\boldsymbol{v}||\boldsymbol{w}|cos\theta)^2$$

$$= |\boldsymbol{v}|^2|\boldsymbol{w}|^2 - |\boldsymbol{v}|^2|\boldsymbol{w}|^2\cos^2\theta$$

$$= |\boldsymbol{v}|^2|\boldsymbol{w}|^2 (1 - \cos^2\theta)$$

$\sin^2\theta + \cos^2\theta = 1$

$\sin^2\theta = 1 - \cos^2\theta$

$$= |\boldsymbol{v}|^2|\boldsymbol{w}|^2 \sin^2\theta$$

$$= (|\boldsymbol{v}||\boldsymbol{w}|sin\theta)^2$$

$|\boldsymbol{v} \times \boldsymbol{w}| = |\boldsymbol{v}||\boldsymbol{w}|sin\theta$

$$= |\boldsymbol{v} \times \boldsymbol{w}|^2$$

$$|\boldsymbol{v}|^2|\boldsymbol{w}|^2 - (\boldsymbol{v} \cdot \boldsymbol{w})^2 = |\boldsymbol{v} \times \boldsymbol{w}|^2$$