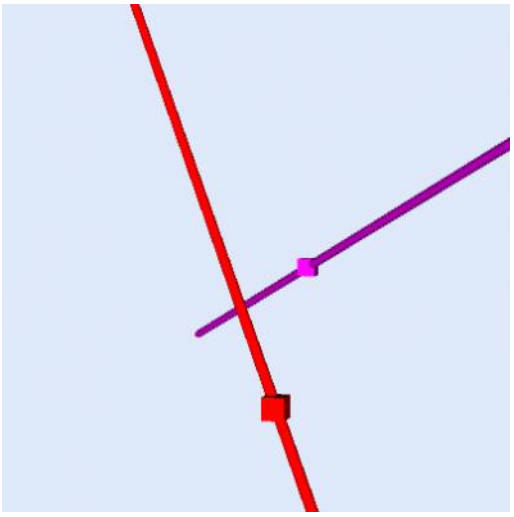
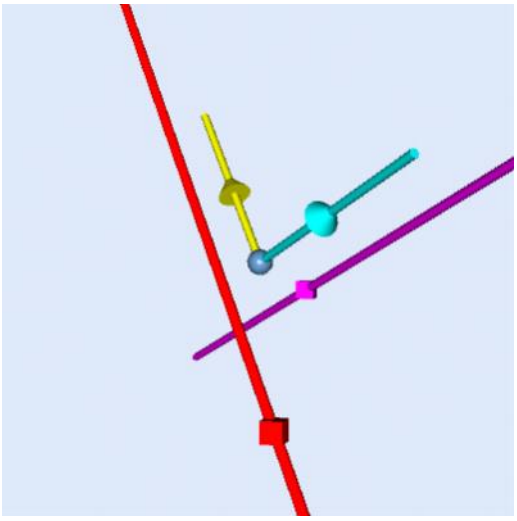
Angle between 2 lines

Find the angle between the lines

$$L_1 : \frac{x-1}{-1} = -y = \frac{z+2}{\sqrt{3}}$$

$$L_2 : \frac{x+2}{2} = \frac{2y+1}{4} = z+1$$





Angle between **lines** = Angle between **direction vectors** parallel to the lines

We can work out the direction vectors from the equations of the lines

$$\frac{x-x_0}{l} = \frac{y-y_0}{m} = \frac{z-z_0}{n} \qquad \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x_0 \\ y_0 \\ z_0 \end{pmatrix} + \lambda \begin{pmatrix} l \\ m \\ n \end{pmatrix}$$

Direction vector = $\begin{pmatrix} l \\ m \\ n \end{pmatrix}$

Find the direction of the lines

$$L_1 : \frac{x-1}{-1} = -y = \frac{z+2}{\sqrt{3}} = \lambda$$

$$\frac{x-1}{-1} = \lambda \qquad -y = \lambda \qquad \frac{z+2}{\sqrt{3}} = \lambda$$

$$x - 1 = -\lambda \qquad y = -\lambda \qquad z + 2 = \sqrt{3}\lambda$$

$$x = 1 - \lambda \qquad y = -\lambda \qquad z = -2 + \sqrt{3}\lambda$$

Write in vector form

$$L_1 : \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ -2 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ -1 \\ \sqrt{3} \end{pmatrix}$$

Direction vector = $\begin{pmatrix} -1 \\ -1 \\ \sqrt{3} \end{pmatrix}$

$$L_2 : \frac{x+2}{2} = \frac{2y+1}{4} = z+1 = \mu$$

$$\frac{x+2}{2} = \mu \qquad \frac{2y+1}{4} = \mu \qquad z+1 = \mu$$
$$x+2 = 2\mu \qquad 2y+1 = 4\mu \qquad z = -1+\mu$$
$$x = -2+2\mu \qquad 2y = -1+4\mu$$
$$y = \frac{-1}{2} + 2\mu$$

Write in vector form

$$L_2 : \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -2 \\ -0.5 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix}$$

Direction vector = $\begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix}$

Find angle between two direction vectors

$\begin{pmatrix} -1 \\ -1 \\ \sqrt{3} \end{pmatrix}$ *and* $\begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix}$

Angle between 2 vectors **a** and **b**

$$\boldsymbol{cos\theta} = \frac{\boldsymbol{a} \cdot \boldsymbol{b}}{|\boldsymbol{a}||\boldsymbol{b}|}$$

$$\begin{pmatrix} -1 \\ -1 \\ \sqrt{3} \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix} = -1 \cdot 2 + (-1) \cdot 2 + \sqrt{3} \cdot 1 = -4 + \sqrt{3}$$

$$\left| \begin{pmatrix} -1 \\ -1 \\ \sqrt{3} \end{pmatrix} \right| = \sqrt{(-1)^2 + (-1)^2 + (\sqrt{3})^2} = \sqrt{5}$$

$$\left| \begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix} \right| = \sqrt{2^2 + 2^2 + 1^2} = \sqrt{9} = 3$$

$$\boldsymbol{cos\theta} = \frac{-4+\sqrt{3}}{3\sqrt{5}}$$

$$\boldsymbol{\theta \approx 110° \; or \; \theta \approx 70°}$$