Scalar Product and Angle between Vectors

In 2 dimensions

$$\boldsymbol{v} = \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} \qquad \boldsymbol{w} = \begin{pmatrix} w_1 \\ w_2 \end{pmatrix}$$

$$\boldsymbol{v} \cdot \boldsymbol{w} = \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} \cdot \begin{pmatrix} w_1 \\ w_2 \end{pmatrix} = v_1 \cdot w_1 + v_2 \cdot w_2$$

In 3 dimensions

$$\boldsymbol{v} = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} \qquad \boldsymbol{w} = \begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix}$$

$$\boldsymbol{v} \cdot \boldsymbol{w} = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} \cdot \begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix} = v_1 \cdot w_1 + v_2 \cdot w_2 + v_3 \cdot w_3$$

Angle between 2 vectors $\boldsymbol{v}$ and $\boldsymbol{w}$

$$\boldsymbol{cos\theta} = \frac{\boldsymbol{v} \cdot \boldsymbol{w}}{|\boldsymbol{v}||\boldsymbol{w}|}$$

Useful Result

When 2 vectors are perpendicular

$$\boldsymbol{v} \cdot \boldsymbol{w} = 0$$

Find angle between two direction vectors $\begin{pmatrix} 1 \\ 2 \\ \sqrt{3} \end{pmatrix}$ *and* $\begin{pmatrix} -1 \\ 3 \\ -2 \end{pmatrix}$

$$\begin{pmatrix} 1 \\ 2 \\ \sqrt{3} \end{pmatrix} \cdot \begin{pmatrix} -1 \\ 3 \\ -2 \end{pmatrix} = 1 \cdot (-1) + 2 \cdot 3 + \sqrt{3} \cdot (-2) = 5 - 2\sqrt{3}$$

$$\left| \begin{pmatrix} 1 \\ 2 \\ \sqrt{3} \end{pmatrix} \right| = \sqrt{1^2 + 2^2 + (\sqrt{3})^2} = \sqrt{8}$$

$$\left| \begin{pmatrix} -1 \\ 3 \\ -2 \end{pmatrix} \right| = \sqrt{(-1)^2 + 3^2 + (-2)^2} = \sqrt{14}$$

Angle between 2 vectors $\boldsymbol{v}$ and $\boldsymbol{w}$

$$\boldsymbol{cos\theta} = \frac{\boldsymbol{v} \cdot \boldsymbol{w}}{|\boldsymbol{v}||\boldsymbol{w}|}$$

$$\boldsymbol{cos\theta} = \frac{5 - 2\sqrt{3}}{\sqrt{8}\sqrt{14}}$$

$\approx 81.7°$ or $98.3°$