IB DIPLOMA PROGRAMME
PROGRAMME DU DIPLÔME DU BI
PROGRAMA DEL DIPLOMA DEL BI

88077401

MATHEMATICAL STUDIES
STANDARD LEVEL
PAPER 1

Monday 5 November 2007 (afternoon)

1 hour 30 minutes

Candidate session number

0	0							

INSTRUCTIONS TO CANDIDATES

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- Answer all the questions in the spaces provided.
- Unless otherwise stated in the question, all numerical answers must be given exactly or to three significant figures.

Maximum marks will be given for correct answers. Where an answer is wrong, some marks may be given for correct method, provided this is shown by written working. Working may be continued below the box, if necessary. Solutions found from a graphic display calculator should be supported by suitable working, e.g. if graphs are used to find a solution, you should sketch these as part of your answer.

1. (a) Calculate $\dfrac{77.2 \times 3^3}{3.60 \times 2^2}$. *[1 mark]*

(b) Express your answer to part (a) in the form $a \times 10^k$, where $1 \le a < 10$ and $k \in \mathbb{Z}$. *[2 marks]*

(c) Juan estimates the length of a carpet to be 12 metres and the width to be 8 metres. He then estimates the area of the carpet.

   (i) Write down his estimated area of the carpet. *[1 mark]*

   When the carpet is accurately measured it is found to have an area of 90 square metres.

   (ii) Calculate the percentage error made by Juan. *[2 marks]*

Working:

Answers:

(a) _____

(b) _____

(c) (i) _____

   (ii) _____

2. The universal set U is the set of integers from 1 to 20 inclusive.

A and B are subsets of U where:
A is the set of even numbers between 7 and 17.
B is the set of multiples of 3.

List the elements of the following sets:

(a) A, *[1 mark]*

(b) B, *[1 mark]*

(c) $A \cup B$, *[2 marks]*

(d) $A \cap B'$. *[2 marks]*

Working:

Answers:

(a) _____

(b) _____

(c) _____

(d) _____

Turn over

3.   The fifth term of an arithmetic sequence is 20 and the twelfth term is 41.

   (a)   (i)   Find the common difference.   *[2 marks]*

   (ii)   Find the first term of the sequence.   *[1 mark]*

   (b)   Calculate the eighty-fourth term.   *[1 mark]*

   (c)   Calculate the sum of the first 200 terms.   *[2 marks]*

Working:

Answers:

(a)   (i)   _____

       (ii)   _____

(b)   _____

(c)   _____

4.   The birth weights, in kilograms, of 27 babies are given in the diagram below.

$$
\begin{array}{c|l}
1 & 7, \ 8, \ 9 \\
2 & 1, \ 2, \ 2, \ 3, \ 5, \ 5, \ 7, \ 8, \ 9 \\
3 & 0, \ 1, \ 3, \ 4, \ 5, \ 5, \ 6, \ 6, \ 7, \ 9 \\
4 & 1, \ 1, \ 2, \ 3, \ 7
\end{array}
\qquad \text{key } 1|7 = 1.7 \text{ kg}
$$

(a)   Calculate the mean birth weight.   *[2 marks]*

(b)   Write down:

    (i)   the median weight,   *[1 mark]*

    (ii)   the upper quartile.   *[1 mark]*

The lower quartile is 2.3 kg.

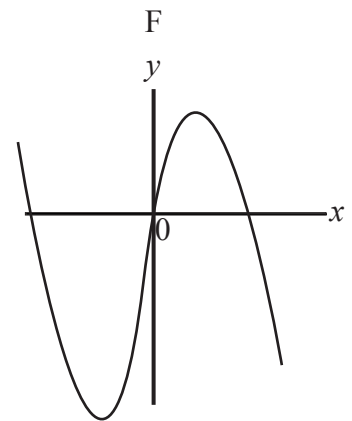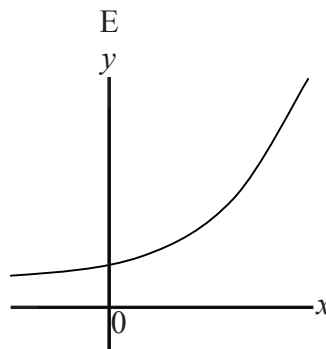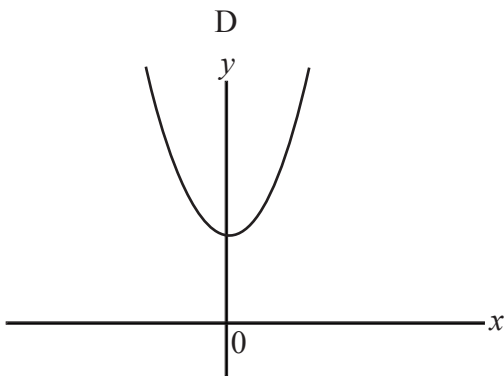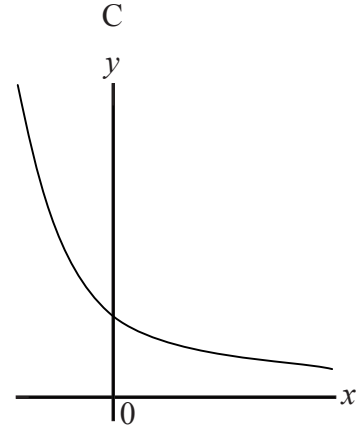(c)   On the scale below draw a box and whisker diagram to represent the birth weights.   *[2 marks]*
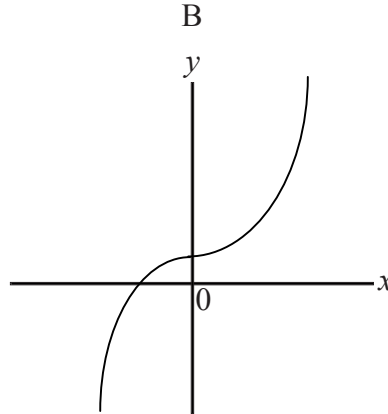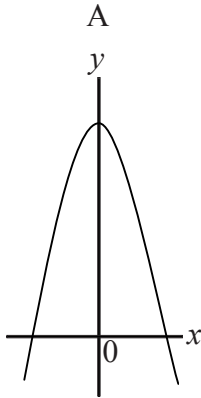
Weight (kg)

Working:

Answers:

(a)   _____

(b)   (i)   _____

    (ii)   _____

5. The following curves are sketches of the graphs of the functions given below, but in a different order. Using your graphic display calculator, match the equations to the curves, writing your answers in the table below.

(the diagrams are not to scale)

A

B

C

D

E

F

	Function	Graph label
(i)	$y = x^3 + 1$	
(ii)	$y = x^2 + 3$	
(iii)	$y = 4 - x^2$	
(iv)	$y = 2^x + 1$	
(v)	$y = 3^{-x} + 1$	
(vi)	$y = 8x - 2x^2 - x^3$	

[6 marks]

6. Two brothers Adam and Ben each inherit $6500. Adam invests his money in a bond that pays simple interest at a rate of 5% per annum. Ben invests his money in a bank that pays compound interest at a rate of 4.5% per annum.

(a) Calculate the value of **Adam's** investment at the end of 6 years. *[3 marks]*

(b) Calculate the value of **Ben's** investment at the end of 6 years. Give your answer **correct to 2 decimal places**. *[3 marks]*

Working:

Answers:

(a) _____

(b) _____

7.   You may choose from three courses on a lunchtime menu at a restaurant.

s: you choose a salad,
m: you choose a meat dish (main course),
d: you choose a dessert.

You choose a **two** course meal which **must** include a main course and either a salad or a dessert, but not both.

(a)   Write the sentence above using logic symbols.   *[2 marks]*

(b)   Write in words $s \Rightarrow \neg d$.   *[2 marks]*

(c)   Complete the following truth table.   *[2 marks]*

s	d	$\neg s$	$\neg s \Rightarrow d$
T	T		
T	F		
F	T		
F	F		

Working:

Answers:

(a)   _____

(b)   _____

_____

_____

8. A school offers three activities, basketball (*B*), choir (*C*) and drama (*D*). Every student must participate in at least one activity.
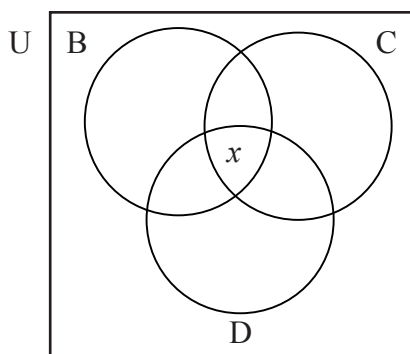
> 16 students play basketball only.
> 18 students play basketball and sing in the choir but do not do drama.
> 34 students play basketball and do drama but do not sing in the choir.
> 27 students are in the choir and do drama but do not play basketball.

(a) Enter the above information on the Venn diagram below.   *[2 marks]*

99 of the students play basketball, 88 sing in the choir and 110 do drama.

(b) Calculate the number of students *x* participating in all three activities.   *[1 mark]*

(c) Calculate the total number of students in the school.   *[3 marks]*
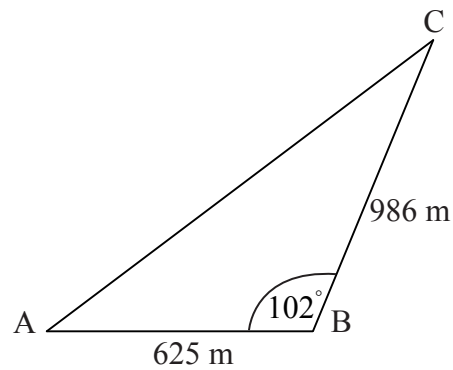
Working:

Answers:

(b) _____

(c) _____

9. On a map three schools A, B and C are situated as shown in the diagram.

Schools A and B are 625 metres apart.
Angle ABC = 102° and BC = 986 metres.

(a) Find the distance between A and C. *[3 marks]*

(b) Find the size of angle BAC. *[3 marks]*

Working:

Answers:

(a) _____

(b) _____

10. (a)   Solve the following equation for x

$$3(2x+1)-2(3-x)=13.$$ *[2 marks]*

(b)   Factorize the expression $x^2 +2x-3$.   *[2 marks]*

(c)   Find the **positive** solution of the equation

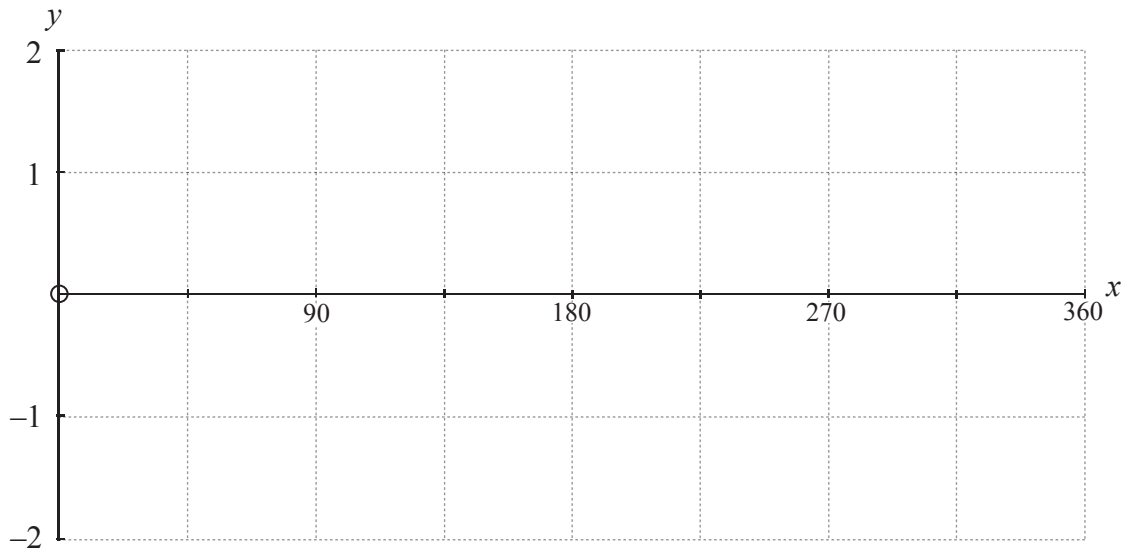$$x^2 +2x-6=0.$$ *[2 marks]*

Working:

Answers:

(a) _____

(b) _____

(c) _____

11. (a) Sketch the graph of the function $y = 1 + \dfrac{\sin(2x)}{2}$ for $0° \le x \le 360°$ on the axes below.   *[4 marks]*

(b) Write down the period of the function.   *[1 mark]*

(c) Write down the amplitude of the function.   *[1 mark]*
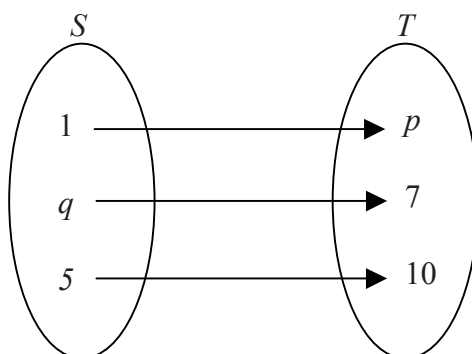
Working:

Answers:

(b) _____

(c) _____

12. (a) $f : x \rightarrow 3x - 5$ is a mapping from the set S to the set T as shown below.

Find the values of p and q.   *[2 marks]*

(b)   A function g is such that $g(x) = \dfrac{3}{(x-2)^2}$.

   (i)   State the domain of the function $g(x)$.   *[2 marks]*

   (ii)   State the range of the function $g(x)$.   *[1 mark]*

   (iii)   Write down the equation of the vertical asymptote.   *[1 mark]*

Working:

Answers:

(a) _____

(b)   (i) _____

       (ii) _____

       (iii) _____

1316

Turn over

13. The mid-point, M, of the line joining $A(s, 8)$ to $B(-2, t)$ has coordinates $M(2, 3)$.

(a) Calculate the values of s and t. *[2 marks]*

(b) Find the equation of the straight line perpendicular to AB, passing through the point M. *[4 marks]*

Working:

Answers:

(a) _____

(b) _____

1416

14. Tania wishes to see whether there is any correlation between a person's age and the number of objects on a tray which could be remembered after looking at them for a certain time.

She obtains the following table of results.

Age (x years)	15	21	36	40	44	55
Number of objects remembered (y)	17	20	15	16	17	12

(a) Use your graphic display calculator to find the equation of the regression line of y on x. *[2 marks]*

(b) Use your equation to estimate the number of objects remembered by a person aged 28 years. *[1 mark]*

(c) Use your graphic display calculator to find the correlation coefficient r. *[1 mark]*

(d) Comment on your value for r. *[2 marks]*

Working:

Answers:

(a) _____

(b) _____

(c) _____

(d) _____

15. A function is represented by the equation

$$f(x) = ax^2 + \frac{4}{x} - 3 .$$

(a)   Find $f'(x)$. [3 marks]

The function $f(x)$ has a local maximum at the point where $x = -1$.

(b)   Find the value of a. [3 marks]

Working:

Answers:

(a) _____

(b) _____

1616